# Understanding Projectile Acceleration 

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#### Abstract

Throwing and catching balls or other objects is a generally highly practiced skill; however, conceptual as well as perceptual understanding of the mechanics that underlie this skill is surprisingly poor. In 5 experiments, we investigated conceptual and perceptual understanding of simple ballistic motion. Paper-and-pencil tests revealed that up to half of all participants mistakenly believed that a ball would continue to accelerate after it left the thrower's hand. Observers also showed a remarkable tolerance for anomalous trajectory shapes. Perceptual judgments based on graphics animations replicated these erroneous beliefs for shallow release angles. Observers' tolerance for anomalies tended to decrease with their distance from the actor. The findings are at odds with claims of the naive physics literature that liken intuitive understanding to Aristotelian or medieval physics theories. Instead, observers seem to project their intentions to the ball itself (extemalization) or even feel that they have power over the ball when it is still close.


People's explicit intuitive knowledge about the dynamics of moving objects is often erroneous (Pittenger, 1991; Proffitt \& Gilden, 1989; Shanon, 1976). Likewise, people's perceptual knowledge about these events is far from perfect (Kaiser, Proffitt, Whelan, \& Hecht, 1992). One of the prime examples that has been studied in this area, known as "intuitive physics," is projectile motion. Cognitive, perceptual, and developmental aspects of falling objects have been studied (Caramazza, McCloskey, \& Green, 1981; Clement, 1982; Kaiser, Jonides, \& Alexander, 1986; Kaiser, Proffitt, \& McCloskey, 1985; Krist, Fieberg, \& Wilkening, 1993; McCloskey, Washburn, \& Felch, 1983). On the basis of motion trajectories that observers predict, perceive, or reproduce, a straight-down belief similar to medieval impetus theory has been proposed to guide people's intuitive understanding of projectile motion (McCloskey et al., 1983). In this article, we concentrate on a neglected aspect of projectile motion, namely on ballistic motion. We show that the impression of acceleration and of maximal object velocity in this case is subject to surprising misconceptions. Just as the perceived trajectory of the motions can be used to draw conclusions about the understanding of Newtonian physics, so can judgments of maximal velocity along the projectile's trajectory.

We compared explicit and implicit understanding of the

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velocity change over time that should occur in an object that is propelled forward and upward at the same time, such as a baseball hit by a batter or a cannonball fired from a barrel. Some researchers have concluded that observers know about the downward acceleration of objects and somehow conceive of it as replacing-not combining additively-with the object's velocity before the onset of the accelerating force of gravity, as at release of carried objects (e.g., Krist et al., 1993). We investigated moving projectiles and found that objects were expected to accelerate long after they left the hand of the thrower or the cannon barrel. This suggests that even more basic principles of physics than Newtonian mechanics are violated in people's conceptual and perceptual knowledge. We argue that the externalization of body mechanics might explain conceptual and perceptual errors.

## Naive Physics, Impetus Theory, and Projectile Trajectories

Unlike conceptions about momentum or impetus, beliefs about the shape of ballistic trajectories have hardly changed from Aristotle's times through the Middle Ages. It was not until the early 17 th century that Galilei suggested the parabolic shape (Wunderlich, 1977). A revealing document is the illustration of ballistic trajectories devised by Paulus Puchner in 1577 and used to instruct cannoneers of the Saxonian artillery, as shown in Figure 1. Puchner's state-of-the-art prediction of cannon ball trajectories and distances was based on the Aristotelian notion of a three-step flight path (Wunderlich, 1977). A straight ascension phase was followed by a circular arc phase and finally by a straight vertical drop. To calibrate a cannon for a given amount and density of gunpowder as well as a given projectile made of wood or metal, first a shot had to be fired at a gun barrel angle of $45^{\circ}$ to determine the maximum reach of the cannon, which was 850 m for this example. Then, a square was constructed (in the drawing it is mistakenly rectangular) with the cannon at its bottom right corner. For any desired horizontal shot length, the corresponding barrel angle was

Figure 1. Ballistic trajectories as devised by Paulus Puchner in 1577. His state-of-the-art prediction of cannonball shot distances was based on the Aristotelian notion of a three-step flight path: first a straight ascension path, second a ciruular arc, and finally a striight vertical drop. Reprinted with permission of the Staaticher Mathematisch-Physikalischer Salon, Dresdener Zwinger, Dresden, Germany.
now determined by drawing a vertical line from the target position to the top of the square. The cannon had to be aimed at that point. To adjust the cannon accordingly, Puchner constructed a special plum bob device. Apparently, this technique provided adequate accuracy at the time.

This illustration of the three-step trajectory leaves unanswered the question of the forces that Puchner thought would act on the ball, because the circular phase of the flight path cannot be explained by air resistance diminishing the original impetus but only by gravity (or something else) already acting on the cannonball. The last straight-down phase was probably based on the empirical observation that cannonballs tended to hit from almost straight above.

As Puchner and the cannoneers of his time knew, the maximal reach of a cannon is obtained at a launch angle of $45^{\circ}$. In a series of studies, Stimpel (1933) showed that actors do have intuitive access to this knowledge. Observers had to throw an ivory sphere at a target marked on a table at a $5-\mathrm{m}$ distance. An ingenious arrangement of contact wires hidden underneath the cloth surface of the table and next to the participant's hand allowed to measure localization error as well as the duration of a throw. Because all balls were released at the same point 5 m away from the target, distance and duration were sufficient to calculate the launch angle for each throw. Untrained observers first executed inefficient throws; however, after several series of 317 throws, they approached ideal launch angles of $45^{\circ}$. Thus, actors seem to acquire implicit (not explicit) knowledge of the relation between launch angle and energy expenditure and home in on the most efficient trajectory. However, this knowledge has to be generated on the spot. Likewise, children moving on a conveyor belt do not know but learn quickly when to release an object to hit a target on the ground. They always start out by releasing the object right above the target (Krist, Loskill, \& Schwarz, 1996).

The fact that people use such procedural implicit knowledge in throwing is not in conflict with the often grossly inaccurate explicit knowledge about the shape of projectile trajectories. To the contrary, projectiles such as fly balls assume a range of different paths (Brancazio, 1985), and physically possible shapes need not necessarily stand out perceptually compared with impossible paths. To control for errors based on trajectory shape misconceptions, we predetermined trajectories and presented them visually in the first experiments. We then investigated perceptual judgments of projectile trajectories.

## Velocity Components of Projectile Motion

Given the ubiquitous nature of events that involve the throwing of objects, one could expect that people understand the speed changes over time that are associated with such events to a better degree than they understand the shape of trajectories when a sudden force starts to act (McCloskey et al., 1983). A typical throw results in a nearly parabolic trajectory of the object. During the first ascending phase of such an event, the ball's vertical velocity decreases because of the gravitational force. It is zero when the ball has reached its apex. The ball is then accelerated by gravity on the descending part of its trajectory. Its horizontal velocity
decreases steadily by a small amount because of air resistance. Thus, the velocity of a moving projectile can be decomposed into two components: the lateral velocity, which is approximately constant neglecting air resistance, and the vertical component, which is decelerating to zero and then accelerating again. The final vertical velocity component when the ball reaches the catcher is approximately equal in magnitude to the vertical exit velocity. Neglecting air resistance, they would be exactly equal provided that the thrower and catcher release and catch the ball at the same height.

Have people internalized these physical facts to some degree, and, if so, how are these facts reflected in the ability to tell proper velocity changes over time from anomalous ones? To assess intuitive knowledge about these two velocity components, we focused on varying horizontal velocity. Research on the ability to detect and judge vertical acceleration suggests that observers are not very sensitive to this information (Gottsdanker, Frick, \& Lockard, 1961; Hecht, Kaiser, \& Banks, 1996). Observers can, however, detect optical accelerations when average ball velocities are slow (Calderone \& Kaiser, 1989, used velocities around $1 \%$ s) or when acceleration does not have to be judged explicitly but is implicitly used to determine where a fly ball will hit the ground (Babler \& Dannemiller, 1993; Michaels \& Oudejans, 1992). A rich stimulus environment also facilitates the detection of gravitational anomalies (Stappers \& Waller, 1993).

For simple and well-anchored horizontal motions, Runeson (1974) has shown that uniform linear particle motions are not perceived as such. They appear to speed up at the beginning and to slow down toward the end of the motion sequence. This effect can be ameliorated when lead-in motion is shown or when objects do not start from a resting position. Likewise, velocity changes are best detected when the transition between them is not smooth but abrupt or if the transition phase is occluded (Schmerler, 1976).

From these findings, several hypotheses about observers' understanding of vertical and horizontal velocity components of projectiles can be made. First, observers should be more likely to be wrong about vertical acceleration than about horizontal velocity. However, this presupposes that they properly distinguish between the two components. In Experiment 4 we show that many observers do not. Second, depending on how well acceleration or deceleration is detected, observers should predict or perceive the projectile either to move with constant speed or to decelerate, then accelerate. If air resistance is factored in, they might also conceive of it as continuously decelerating. Accordingly, our first experiment was designed to test these predictions and to assess cognitive understanding of the velocity changes during the trajectory of a fly ball. Astonishingly, many participants believed that the ball would accelerate during the ascending phase after it had left the thrower's hand. In Experiments 2 and 3 we elaborated on these misconceptions. Results of the fourth experiment, in which we presented observers with animated versions of various ball throws, confirm the "cognitive" bias to be mirrored in visual perception. Finally, in Experiment 5, observers were pre-


Figure 2. In Experiment 1, each participant saw either the straight (dashed line) or the curved trajectory (solid line). He or she was asked to consider the path of a ball thrown from a player (on the left) to a catcher (on the right) and to mark the point where the ball would have maximal speed.
sented with a 3-D animation of throws that receded into depth.

## Experiment 1: Magical Acceleration

Throughout evolution, and throughout development, the Newtonian laws of projectile motion could have been internalized to some approximation (see Shepard, 1994). Depending on the degree of approximation, Aristotelian conceptions (Shanon, 1976), medieval impetus theory (McCloskey et al., 1983), and heuristic principles (Kaiser et al., 1992) have been suggested to describe intuitive reasoning about projectile motion. Most of these can be thought of as the exclusive (or sequential) focusing on one parameter such as the effectiveness of air resistance or gravity. The present experiments were designed to assess whether these hypotheses would generalize from trajectory shape to the understanding of velocity changes that the objects undergo as they follow their respective motion trajectories. That is, the following basic question was posed: To what extent do explicit judgments about velocity changes reflect the true state of affairs? Thus, if asked to judge whether a projectile accelerates after it has been released by a thrower, participants should indicate that the object might move at constant speed or decelerate and then accelerate, depending on whether they factor in gravity and drag (air resistance). We tested this by asking students to indicate the point of maximal velocity on the depicted trajectory of a baseball thrown across a large distance.

## Method

Participants. In a mass-testing session at the University of Virginia, 176 undergraduates completed a paper-and-pencil test to partially fulfill a research option in an introductory psychology class.

Stimuli and procedure. Two groups of participants were presented with two versions of a schematic drawing as part of a larger paper-and-pencil questionnaire. In the drawing, a person (stick figure) was shown throwing a ball to another person, as can be seen in Figure 2. The two stick figures stood on a flat terrain (horizontal line). In the first version of the drawing, a symmetrical curved trajectory was drawn to indicate the trajectory on which the ball would travel, as depicted by the solid line. A short paragraph explained that the trajectory indicated the approximate flight path. In the second version, the trajectory was a straight path, as indicated by the dashed line. The curved trajectory was presented to 84 participants and the straight one to 92 participants. They picked
their questionnaire from a pile consisting of alternating blocks; thus, the assignment was pseudorandom.
Participants were asked to consider the ball thrown from the player on the left to the catcher (on the right) and to mark the point where the ball would have maximal speed by drawing a cross directly on that part of the trajectory. On a second page of the questionnaire, participants were asked to choose one of multiple options to explain what they thought would happen to the ball's speed during the flight from the thrower to the catcher. The categories were as follows:

Category A: The ball moves at constant speed throughout its path.

Category B: The ball continuously speeds up.
Category C: The ball continuously slows down.
Category D: The ball first speeds up then slows down.
Category E: The ball first slows down then speeds up.
Category F: Other, please describe.
Note that for the curved trajectories, only Category E was compatible with the ball's true behavior. For straight trajectories that approximated a fast throw, Categories A and C were valid depending on whether air resistance was considered.

## Results

The marked positions of maximal velocity along the projectile trajectory were coded with a numeric value between 1 and 5, depending on the category into which they fell. As can be seen in Figure 3, ascending numbers represent a progression along the trajectory. Vertical dotted lines indicate category boundaries. This coding tumed out to be practical because many participants drew circles or marked the position imprecisely. Answers were averaged separately for curved and straight trajectories. Subjective positions of maximal speed are marked by a cross on each trajectory in Figure 3. Surprisingly, the subjective points of maximal velocity did not coincide with the release point that was the correct answer. Neither did they reflect the final point of the curve at the catcher's hand, which would also be plausible when neglecting air resistance. Participants believed that maximal velocity was reached close to the middle of the trajectory. The average subjective maximum for the curved trajectory tended to be even closer to the apex than it was for the straight path, $t(174)=21.14, p<.001$.


Figure 3. Answers were averaged according to the numbers of the category into which they fell, with ascending numbers representing progression along the trajectory. Vertical dotted lines indicate category boundaries. Averaged positions of maximal speed are marked by a cross separately for the curved and straight trajectories. Data from Experiment 1 (crosses) and Experiment 3A (open circles) are presented.

Table 1 shows the categorical choices participants made after they had identified the point of maximal velocity. The distribution corresponds to their initial response: Balls that accelerate after they leave the thrower's hand (Categories B and D) were preferred and judged as most natural by $47.2 \%$ of all participants. A closer look at the difference between the trajectory types revealed that participants distinguished them in Categories C and E, $\chi^{2}(5, N=176)=39.15, p<$ .001. Straight trajectories were more often associated with continuous deceleration. Curved trajectories, on the other hand, were more often associated with initial deceleration and subsequent acceleration. These data reflect some correct knowledge about the vertical velocity component.

## Discussion

Evidence for the belief that projectiles (continue to) accelerate after they leave the hand was found. About one half of all participants expressed this erroneous belief. Could participants have misunderstood the question? One possible misunderstanding might have been that instead of the size of the velocity vector, participants judged only the horizontal component of the ball's movement. However, in this case, the answer should have been that the ball slows down continuously if air resistance is factored in. This answer was chosen by $23.3 \%$ of the participants. If air resistance is neglected, the ball should move with constant horizontal speed; this option was chosen by only $9.1 \%$. Thus, it is impossible that participants generally mistook the horizontal velocity component for the ball's overall speed, although some might have done so. Most participants ( $36.9 \%$ ) thought that the ball would first speed up and then slow down, which is inconsistent with all physically correct interpretations of the event. This provides evidence for misconceptions about projectile motion that go far beyond anything that can be accommodated within the frame of Newtonian, Aristotelian, or medieval theories of physics. The results also seem to be consistent with beliefs throughout individual cognitive development that one can throw farther when aiming at a shallower angle (Krist, 1992). Children of various ages made this mistake when judging the distance that a ping-pong ball would travel if launched at different angles. Likewise, our participants thought that the

Table 1
Frequency Count (and Percentage) of Scenarios Chosen by Participants Who Were Presented With Curved and Straight Trajectories, Respectively, in Experiment 1

| Answer category | Curve | Straight | Total |
| :--- | :---: | :---: | :---: |
| A. Constant speed | $11(13.1)$ | $5(5.4)$ | $16(9.1)$ |
| B. Continuous speeding up | $6(7.1)$ | $12(13.0)$ | $18(10.2)$ |
| C. Continuous slowing down | $7(8.3)$ | $34(36.9)$ | $41(23.3)$ |
| D. First speeding up then | $31(36.9)$ | $34(36.9)$ | $65(36.9)$ |
| slowing down | $37(32.1)$ | $4(4.3)$ | $31(17.6)$ |
| E. First slowing down then |  | $3(3.3)$ | $5(2.8)$ |
| speeding up | $2(2.4)$ | $3(100.0)$ | $92(100.0)$ |
| F. Other | $176(100.0)$ |  |  |
| $\quad$ Total | $84(100.0$ |  |  |

maximal velocity would be reached sooner for shallow angles and thus that the average speed should be faster.

## Experiment 2: Tolerance for Deformed Trajectories

The surprising results of Experiment 1 may reflect the belief that objects continue to accelerate after they leave the actor's hand. However, before we can draw this extreme conclusion, we have to rule out that the results are the artifact of a combination of two factors. First, participants may have noticed that the curved and straight trajectories that they had to consider are, strictly speaking, impossible events. It may have appeared strange that a ball should stay on a perfectly linear or circular path. To render these geometrically simple paths plausible, some additional external force might have been assumed despite the instructions that explicitly ruled out this possibility. The additional force could not only have kept the ball on course but also accelerated it. Second, the multiple-choice question always followed the first question that involved the peak velocity marking. Thus, the latter may have been biased by the former. Participants may have rationalized their qualitative answers to be congruent with the initial mark. The first issue was ruled out in Experiment 2 and the second in Experiment 3.

Researchers know little about what kind of trajectories people believe that a thrown object can follow. In Experiment 1 we drew a simple, shallow circular line and were satisfied that it would look reasonable to most observers. Naturally occurring ballistic trajectories can deviate more or less from the parabolic shape found in a vacuum. If the ball is relatively light and its surface irregular, striking deviations from the parabolic trajectory are possible (e.g., Adair, 1990). Thus, a test of how much a trajectory can deviate from a naturally occurring one until it is considered unnatural is problematic. Presumably, the tolerance for odd shapes is high, but, on the other hand, actors have some implicit knowledge about trajectory shape (Krist et al., 1993). Experiment 2 was designed to answer the question of whether the circular and straight trajectory shapes that we used stood out in any way that would lead us to believe that trajectory shape was responsible for the results. We used a paper-and-pencil questionnaire similar to the one used in Experiment 1, but we asked participants to compare 12 different trajectory shapes. Because a clear standard of what constitutes a natural trajectory is missing, we complemented the circular and straight paths used before with examples of the basic conic functions (e.g., parabola, hyperbola, ellipse) plus a sine function, a linear function, and a trajectory constructed according to the instructions of Puchner. We also added a deformed version of all trajectories that could be perceived to reflect wind effects.

## Method

Participants. Ninety-six undergraduates from six cognitive psychology classes at Staffordshire University completed a onepage paper-and-pencil test. Their ages ranged from 19 to 47 years (average $=22.6$ ). They were naive about the purposes of the study.
Stimuli and procedure. The 12 trajectory shapes are shown in Figure 4. This is an example of the test as it was presented to the students. Moreover, four different random orders on the page of the


Figure 4. Trajectory shapes used in Experiment 2: circular, linear, sine (from 0 to $\pi$ ), hyperbolic, parabolic, and Puchner (linear, then circular, then vertical). Trajectories shown on the right side of the figure roughly approximated air effects. These deformed trajectories were generated from the original curves (those shown on the left side of the figure, excluding Puchner) by scaling the first half horizontally to $150 \%$ and the second half to $50 \%$. Finally, a straight trajectory (bottom right comer of figure) was used.

12 drawings were used. In each drawing a person on the left (stick figure) was shown throwing a ball to another person on the right. The task was to rate how natural the shape of the trajectory looked. The following instructions were printed on the top of the page: "The person on the left throws a ball to the person on the right. They are on a flat ground and there is no wind. The solid line represents the trajectory. Please judge on a scale from 1 to 10 whether the trajectory looks natural and realistic to you. ( $1=$ absolutely impossible $10=$ very natural)." Participants were told that any way of throwing the ball by the thrower was acceptable. Information was also collected about gender, age, and sports that the students regularly practiced.
In the first column of Figure 4, the trajectories have the following shapes: circular, linear, sine (from 0 to $\pi$ ), hyperbolic, parabolic, and Puchner (linear, then circular, then vertical). It was of particular concern to establish how observers would treat deformations of these symmetrical shapes. Drag and head wind effects can produce paths that are neither parabolic nor symmetrical. These effects can be dramatic. For instance, the force of drag can shorten the range of a throw by up to $40 \%$ (Brancazio, 1985). Drag increases geometrically with object velocity. Also, the steeper the launch angle (up to $45^{\circ}$ ), the stronger the effect of drag. Brancazio calculated that a baseball with a release speed of 60 mph and a launch angle of $20^{\circ}$ would travel in the air about $90 \%$ of its range in a vacuum.

Doubling the exit speed would shorten this range to about $60 \%$. Launch speeds between 60 and 120 mph are within the ability range of professional baseball pitchers (Griffing, 1987).

Thus, except for the Puchner trajectory, we complemented all paths with deformed versions that could reflect wind effects. If noticeable, this manipulation might explain some of the misconceptions. To make the deformations salient, we generated the Paths on the right side of Figure 4 from the original Curves on the left side according to the following process: The curve was split in half, and the first half was scaled horizontally to $150 \%$ and the second half horizontally to $50 \%$. This process produces curves that are still smooth and are composites of two halves that still belong to the original category (e.g., a parabola). Finally, the last drawing was a simple straight trajectory.

## Results

Figure 5 shows the average naturalness ratings for the 12 paths that were presented. Two separate, repeated measures analyses of variance (ANOVAs) were conducted on the dependent measure of judged naturalness. First, all paths that had deformed complements were compared in one analysis (i.e., circular, bent line, sine, hyperbolic, and parabolic). The Puchner trajectory was not added here because the underlying impetus theory did not rely on air resistance; also, an additional compression of that trajectory would not have changed the salient aspects of its shape. The effect of trajectory deformation was not significant, $F(1,95)=2.73, p=.102$. A strong main effect of trajectory shape was found, $F(4,380)=82.36, p<.0001$. Parabolas and sine paths did not differ in their naturalness ratings, but parabolas looked more natural than hyperbolas, $F(1,95)=$ $117.41, p<.0001$, and circular paths, $F(1,95)=7.38, p=$ .0079 . The circular paths looked less natural than sinusoidal and parabolic paths but more natural than all other paths ( $288.0>F>7.38$ ). Linear trajectories were judged significantly lower than all others ( $511.4>F>172.2$ ). There was also a significant interaction between trajectory shape and deformation, $F(4,376)=10.55, p=.0001$. As visible in Figure 5, circular and hyperbolic curves looked more natural when compressed toward their end, whereas the sine trajectory was judged to be less natural in this case.

Gender and experience with ball sports had no main effects, nor did they interact with other independent variables. Students were sorted into two groups on the basis of experience: those who reported regularly playing a ball sport and those who did not or who reported practicing other sports such as swimming. In a second ANOVA, all trajectories without added air effects were compared (i.e., circular, bent line, sine, hyperbolic, parabolic, Puchner, and straight line). A large main effect for trajectory shape was found, $F(6,570)=71.99, p<.0001$. The Puchner trajectory was judged to be more natural than the bent line, $F(1,95)=$ $21.20, p<.0001$, as natural as the hyperbolic curve, and less natural than all other paths, ranging from $F(1,95)=137.69$ to $6.19, p<.02$ to .0001 . The sine wave was judged to be marginally more natural than the parabola, $F(1,95)=4.28$, $p=.0413$. This suggests that the smoothness of the trajectory was a criterion.

Judged Naturalness


Figure 5. Average judged naturalness for all trajectories that were presented within participants in Experiment 2. The scale used had a minimum value of 1 and a maximum value of 10 . Error bars indicate the standard errors of the mean.

## Discussion

Observers were tolerant and accepted a large variety of trajectory shapes as natural. The parabolic path received high naturalness ratings, but so did circular and sinusoidal paths. Deformation that could be attributed to air effects did not, on average, lead to the impression of more realistic trajectory shape. Symmetrical and skewed curves were both judged to be natural, suggesting that, in some cases, observers did take air resistance into account. Even the blatantly wrong Puchner trajectory received ratings that were far better than the "impossible" score of one. It fared better than other impossible paths, such as the bent line paths with a linear-up path, followed by a linear-down path. Thus, except for the straight line, all trajectories that had curvature looked more or less acceptable, whereas a combination of linear segments looked unnatural. The presentation of the circular path in Experiment 1 is unlikely to have prompted observers to assume acceleration because of the subjective unnaturalness of the presented shape. The large tolerance for different trajectory shapes can also be exploited in animated versions of the task by holding shape constant and systematically varying the speed of the projectile, as was done in Experiment 4.

## Experiment 3A: Replication and the Role of Experience

Because the results obtained in Experiment 1 were so counterintuitive, we wanted to replicate them with a new set
of participants. Also, experience with ball trajectories was not controlled. The false assumption of postrelease acceleration might disappear for observers who were experienced with ball sports. Thus, we replicated Experiment 1, but we chose university students enrolled in different degree programs. In particular, we included students from a sports studies program. These students differed from psychology students in both the level of practical experience in competitive sports and of theoretical knowledge about sports. Comparing these data with data from Experiment 1 should inform us about the role of expertise. We also used a questionnaire to determine the sports the students played and the intensity of their practices.

## Method

Participants. Forty-eight undergraduates ( 25 men and 23 women) at Staffordshire University completed a paper-and-pencil test. They were aged 18-21 years (average $=20$ ). Thirty-six students were enrolled in a sports studies program (BSc), and 12 were from other programs, mostly law (BA/LLB). This subset was included to provide a control of students from the same institution.

Stimuli and procedure. The stimuli and procedure were identical to those used in Experiment 1 (see Figure 2). The curved path was presented to 22 participants and the straight one to 26 participants. All were asked to consider the ball thrown from the thrower to the catcher and to mark the point where the ball would have maximal speed by drawing a cross. On a second page of the questionnaire, participants were asked to choose one of multiple options to explain what they thought would happen to the ball's speed during its flight.

## Results

The marked positions of maximal velocity along the projectile path were coded with a numerical value between 1 and 5. Answers were averaged separately for curved and straight paths. Subjective positions of maximal speed are marked by an open circle on each path in Figure 3. The average positions were remarkably close to those found in Experiment 1 (crosses in Figure 3). Students were classified as ballplayers if they reported playing a ball sport regularly for the past 2 years. The type of path (straight and curved), gender (male and female), degree (sport studies or other, two levels), and ball-playing experience (yes or no, based on self-reports) were entered in an ANOVA. None of these variables reached significance, nor did the variables interact ( $0.12<F<1.20$ ).

Table 2 shows the categorical choices participants made after they had identified the point of maximal velocity. Balls that accelerated after they left the thrower's hand (Categories $B$ and $D$ ) were judged to be the most natural by $39.6 \%$ of all participants. The table shows the same pattern that was observed in Experiment 1. The bias of undue acceleration was marginally stronger for curved trajectories when counting Categories B and D as biased and Categories C and E (C and A) as acceptable for curved (straight) paths, $\chi^{2}(2, N=48)=5.719, p=.057$. This reflects that straight paths were more often associated with continuous deceleration. Curved paths, on the other hand, were more often associated with initial deceleration and subsequent acceleration, which reflects some correct knowledge about the vertical velocity component.

## Discussion

No effect of experience on beliefs about projectile acceleration was found. It is possible that this sample was too small to detect subtle differences between the groups. It does, however, replicate the outcome of Experiment 1 with remarkable accuracy with participants from a different continent, students enrolled in different degree programs, and students with different skill levels.

Again, it is possible that participants judged only the horizontal component of the ball's movement. In this case, the answer should have been that the ball slows down

Table 2
Frequency Count (and Percentage) of Scenarios Chosen by Participants Who Were Presented With Curved and Straight Trajectories, Respectively, in Experiment 3A

| Answer category | Curve | Straight | Total |
| :--- | :---: | :---: | :---: |
| A. Constant speed | $4(18.2)$ | $3(11.5)$ | $7(14.6)$ |
| B. Continuous speeding up | $1(4.5)$ | $1(3.8)$ | $2(4.2)$ |
| C. Continuous slowing down | $4(18.2)$ | $13(50.0)$ | $17(35.4)$ |
| D. First speeding up and then |  |  |  |
| slowing down | $8(36.4)$ | $9(34.6)$ | $17(35.4)$ |
| E. First slowing down and then |  |  |  |
| speeding up | $5(22.7)$ | $0(0.0)$ | $5(10.4)$ |
| F. Other | $0(0.0)$ | $0(0.0)$ | $0(0.0)$ |
| $\quad$ Total | $22(100.0)$ | $26(100.0)$ | $48(100.0)$ |

continuously. This answer was chosen by $35.4 \%$ of the participants. If air resistance is neglected, the ball should move with constant horizontal speed; this option was chosen by only $14.5 \%$. Therefore, it is possible that some participants exclusively judged the horizontal velocity component, although a large proportion ( $35.4 \%$ ) thought that the ball would first speed up and then slow down, which is inconsistent with all physically correct interpretations of the event.

## Experiment 3B: Exclusion of Potential Order and Coding Effects

There was a slight chance that the order of presentation (first the drawing, then the questions) had prompted ad hoc rationalizations of a supposedly unreflected selection of a mistaken point of maximal velocity. The categorical coding that was chosen for practical purposes could also have unduly amplified the effects. Thus, Experiment 1 was replicated with a reverse order of presentation. First, participants had to choose an answer to the question of how the ball's velocity changes; they then had to pencil in the point of maximal velocity. The positions were coded in distance from the release point.

## Method

Participants. Eighty-four psychology students ( 14 men and 70 women) at Staffordshire University completed a paper-and-pencil test. Their average age was 22 years (range $=18-45$ years).

Stimuli and procedure. The stimuli and procedure were identical to those used in Experiment 1 (see Figure 2), except for the order of presentation. Participants were asked to choose from the same list of multiple options to explain what they thought would happen to the ball's speed during the flight from the thrower to the catcher. On the second page of the questionnaire, the curved path was presented to 45 participants and the straight one to 43 participants. They were asked to consider the ball and mark the point where the ball would have maximal speed. They were instructed to put a precise cross such that its center exactly indicated the spot of maximal velocity.

## Results

The marked positions of maximal velocity along the projectile path were measured as follows: For the straight path with a total length of 140 mm , horizontal distance from the thrower was measured in millimeters. For the curved circular path with a total length of 158 mm , the position of the cross was measured in degrees and later transformed to the distance along the path. Distributions of responses are shown on each path in Figure 6 using box plots. The median positions were remarkably close to the averages of Experiments 1 and 3A (see Figure 3). The gray regions indicate the $95 \%$ confidence intervals for the median. The type of path (straight and curved) was entered as a factor in an ANOVA with distance as the dependent variable. To make circular and straight path data comparable, we normalized the position values of the dependent measure to range from 0 to 100. The location of judged maximal acceleration for curved and straight paths did not differ significantly, $F(1,81)=$ 0.44 .


Figure 6. Positions of subjective maximal velocity for both trajectories obtained in Experiment 3B. The shaded area indicates the $95 \%$ confidence interval around the median. The box encases $50 \%$ of the data from the first to the third quartile.

The distribution of responses is depicted in Figure 7. They cluster around the beginning of the trajectory, its center, and the end. Table 3 shows the categorical choices participants made after they had identified the point of maximal velocity. Balls that accelerated after they left the thrower's hand and then slowed down (Category D) were judged to be the most natural by $53.6 \%$ of all participants. In this analysis, categories were not divided by straight and curved paths because the question was asked on the first page of the questionnaire, before participants saw the drawing.

## Discussion

The results clearly show that participants were not prompted to rationalize their markings by giving answers to the multiple-choice question that they would not otherwise have given. To the contrary, changing the presentation order revealed that when the question was asked first (i.e., without the visual aid of a drawing), even more answers were given that indicated the conceptual bias of an early postrelease acceleration. In this experiment, $53.6 \%$ of all participants had this erroneous belief, compared with $36.9 \%$ and $35.4 \%$ in Experiments 1 and 3A, respectively. We now established beyond any reasonable doubt that a substantial proportion of the college


Figure 7. Frequency distribution of responses obtained in Experiment 3B. The distance from the origin (position of thrower on the left) is shown on the abscissa.

Table 3
Frequency Count (and Percentage) of Scenarios Chosen by Participants in Experiment 3B

| Answer category | Frequency (\%) |
| :--- | :---: |
| A. Constant speed | $13(15.5)$ |
| B. Continuous speeding up | $6(7.1)$ |
| C. Continuous slowing down | $13(15.5)$ |
| D. First speeding up and then slowing down | $45(53.6)$ |
| E. First slowing down and then speeding up | $7(8.3)$ |
| F. Other | $0(0.0)$ |
| $\quad$ Total | $84(100.0)$ |

population exhibits the erroneous belief that projectiles continue to accelerate after being released by the thrower.

## Experiment 4: Animation in Two Dimensions

The observed misconceptions about the velocity of propelled objects might reflect a lack of perceptual attunement to these motions. However, the biases could disappear when observers are presented with animated analogs of the predictions they made in the paper-and-pencil task. Such a facilitating influence of visual animation has been demonstrated for exit trajectories of marbles rolled through a C-shaped tube, which was placed flat on a table top. In paper-and-pencil tasks, participants predicted curved exit trajectories compatible with an erroneous impetus theory (McCloskey, Caramazza, \& Green, 1980). However, these trajectories were perceived veridically as unnatural when they continued to curve after exiting the tube in a computer animation (Kaiser et al., 1992). Thus, animated versions of the erroneous explicit beliefs of projectile motion might be easily noticeable as unnatural. On the other hand, observers are tolerant to trajectory variations in thought (Experiment 2) as well as in virtual environments (Stappers, 1997).

Experiment 4 was designed to assess to what extent observers are able to recognize thrown objects with unnatural velocity profiles as such. The belief by many participants that a baseball would speed up after it leaves the thrower's hand can be interpreted in three different ways depending on which velocity component they misconceive. Participants could have erroneous beliefs (a) about the horizontal component, which is affected only by air resistance; (b) about the vertical velocity component, which is affected by the force of gravity and, to a much smaller degree, by air resistance; or (c) about both components.

We decided to test the first possibility that participants were mistaken about the horizontal velocity component. This was plausible because pilot participants had no problems with choice (b), above. They knew that objects, once thrown straight up, would decelerate, stop, and then accelerate as they fell back down. Moreover, in the case of a shallow throw, the vertical component was almost negligible. Nonetheless, the data of Experiment 1 showed that the belief in postrelease acceleration persisted, suggesting that participants focused on the horizontal velocity component. Thus, ball trajectories were simulated that always obeyed the same canonical vertical deceleration and acceleration
neglecting friction. Horizontal velocities, however, were varied to produce a variety of canonical profiles (neglecting and including air resistance) as well as anomalous profiles. In all cases, the trajectories shapes remained parabolic; thus, the extreme cases of linear or Puchner paths were not simulated.

## Method

Participants. Eleven observers ( 6 women and 5 men) participated in the study. All were enrolled as students of various disciplines at the Ludwig-Maximilians-Universität in Munich. All participants had normal or corrected-to-normal vision and were unfamiliar with the purpose of the study. They were paid for their participation.

Apparatus and stimuli. All stimuli were displayed on an Eizo Flexiscan $15-\mathrm{in}$. ( $38.1-\mathrm{cm}$ ) monitor, which was driven by an HP Vectra RS 20 C PC. Resolution was $640 \times 480$ pixels; the update rate of the display and the refresh rate were 60 Hz . Similar to the drawing shown in Figure 2 (without the trajectories), the display consisted of a long horizontal line 2 cm above the bottom of the screen, which symbolized the ground plane. Two stick figures (baseball players) were standing on either end of the line, separated by a distance of 26 cm . The stick figures were 2 cm tall. One player, referred to as the thrower, held a ball (diameter $=0.5 \mathrm{~cm}$ ). The other player (catcher) was empty-handed. The viewing distance was 30 cm , such that the display subtended a visual angle of $36^{\circ}$. The scene simulated two players separated by 23.4 m and positioned on a plane 36 m away from the observer. ${ }^{1}$

The following four variables were fully crossed within participants.

1. Air resistance. Air resistance was either absent or simulated approximatively as a slight horizontal deceleration. In this case, the length of the trajectory was shortened by $20 \%$. The latter events were scaled up to subtend the same visual angle as the others.
2. Speed. Nine horizontal ball velocities were used. The horizontal velocity component (speed) was $14.6 \mathrm{~m} / \mathrm{s}, 11.4 \mathrm{~m} / \mathrm{s}$, and $8.1 \mathrm{~m} / \mathrm{s}$, which corresponded to durations of the throw between 0.8 and 1.6 s . Three conditions consisted of uniformly constant speeds at these three rates. Moreover, six anomalous velocities were created by changing the horizontal speed for part of the trajectory. First, the ball started at $8.1 \mathrm{~m} / \mathrm{s}$, and after $20 \%$ of its course sped up to $14.6 \mathrm{~m} / \mathrm{s}$. The transition from the slow to the faster speed was smoothed. Second, the ball started out fast ( $14.6 \mathrm{~m} / \mathrm{s}$ ), and after $20 \%$ of its course slowed down. Third, the ball moved slowly (8.1 $\mathrm{m} / \mathrm{s}$ ), and for the last $20 \%$ of its course sped up to the fast speed. Fourth, the ball started out fast and slowed down for the last $20 \%$ of the trajectory. Fifth, the ball started and ended slow but moved fast in the middle $40 \%$ of the trajectory. Finally, the ball could start and end fast but move slowly in the middle. The speed-time diagrams for the six nonconstant horizontal velocity profiles are depicted in Figure 8.
3. Launch angles. The launch angles of the ball with respect to the ground were $45^{\circ}$ or $10^{\circ}$. To maintain a constant visual angle of the display while producing two markedly different trajectories, we had to change the gravity coefficient depending on the horizontal velocity component. On average, it was approximately $9.8 \mathrm{~m} / \mathrm{s}^{2}$. For shallow trajectories ( $10^{\circ}$ launch angle), gravity varied between $1.4 \mathrm{~m} / \mathrm{s}^{2}$ and $5.6 \mathrm{~m} / \mathrm{s}^{2}$; for steep trajectories ( $45^{\circ}$ launch angle), the coefficient varied between 7.0 and $28.1 \mathrm{~m} / \mathrm{s}^{2}$. Thus, all trials neglecting air resistance followed two different parabolic paths depending on the launch angle. All trials with air resistance followed somewhat compressed parabolic paths. The variation in gravity coefficients was the price for constant visual angles and constant scaling relations between displays. The former were


Figure 8. Velocity profiles for stimuli presented in Experiment 4. The abscissa represents time and the ordinate horizontal velocity. The short recoil motion of the arm backward is indicated by negative velocity. $\mathrm{t}_{0}=$ the moment in time when the ball is released from the thrower's hand.
varied because participants are known to be insensitive to gravitational acceleration changes (Gottsdanker et al., 1961).
4. Direction. All trials could either start from the left side of the screen and move toward the right or vice versa.

Procedure. Observers were instructed to look at the stationary scene and attend to the stick figure that held the ball. The latter would be thrown from that player to the one on the opposite side of the screen. A key had to be pressed to initiate the throw. A short recoiling motion preceded the flight phase of the ball. Without such a lead-in, the motions looked choppy and unnatural, which is consistent with findings by Gottsdanker et al. (1961) that lead-in facilitated accurate perception of acceleration.

Observers were encouraged to play the sequence again before delineating a portion of the trajectory that looked strange. On

[^0]average, they repeated the display two to three times per trial. After the throw, the two players remained visible, and a thin line was drawn to indicate the path that the ball had taken. With two mouse clicks, an area of anomaly on this path had to be encased. This measure should assess whether observers were able to spot anomalous velocity transitions. The display was then replaced by a naturalness scale, ranging from 0 (unnatural/strange) to 12 (natural). A rating had to be made by moving the mouse to the appropriate slot.

The experimenter mentioned that the thrower was well trained and had no trouble throwing the comparatively large ball (diameter $=0.5 \mathrm{~cm}$, or $25 \%$ of the thrower's height). After a practice session of about 15 trials, the experimenter encouraged observers to make use of the whole rating scale and to rely on their intuitive judgment. They were also told that they should pay particular attention to the ball's speed. After the experiment, the experimenter asked observers to describe what a natural throw should look like and what, if any, anomalies they encountered in the displays.

## Results

When answering the question about the behavior of a natural throw, 4 observers correctly stated that the ball should have its maximal velocity at the beginning of its trajectory. Four observers thought that maximal velocity was reached after about one third of the trajectory, and 3 observers were convinced that maximal velocity was at the end close to the catcher. These convictions roughly correspond to the findings of Experiment 1.

The localization measure did not reveal any obvious locations of anomalies. Furthermore, many reported to be uncomfortable with the task and had to be told not to worry about it. Thus, this measure was not analyzed further. The naturalness ratings, which observers found intuitive to give, were entered into a repeated measures ANOVA ( 2 air resistances $\times 9$ anomalies types $\times 2$ launch angles $\times 2$ directions). Overall, trials that accelerated after the ball had left the hand were judged to be natural. In addition, the mirror reversal of this situation, balls decelerating toward the end of their trajectories, received maximum ratings. Ball trajectories with a $45^{\circ}$ launch angle looked more realistic than those with a $10^{\circ}$ angle, $F(1,10)=6.68, p=.027$. The former received average ratings of 5.22 and the latter 3.98. A highly significant interaction of launch angle and velocity profile was also found, $F(6,60)=8.12, p<.001$. Figure 9 depicts naturalness ratings separated by launch angle of the ball and velocity profile throughout the trajectory. For this graph, we averaged all three constant horizontal velocities. The manipulation of leftward versus rightward throw showed no significant effect, nor did the manipulation of air resistance.

The repeated measures ANOVA revealed the following contrast effects. Regardless of launch angle, early accelerating trials were not judged to be more natural than constant velocity ones, but they appeared more natural than late accelerating, $F(1,10)=28.04, p<.001$, middle fast, $F(1$,

## Judged Naturalness



Figure 9. Average judged naturalness of different velocity profiles by launch angle in Experiment 4. The scale used had a minimum value of 0 and a maximum value of 12 . Error bars indicate the standard errors of the mean.
$10)=30.71, p<.001$, and middle slow, $F(1,10)=25.94$, $p<.001$, trials. The interaction between launch angle and velocity profile showed significant contrasts between early acceleration and constant velocity, $F(1,10)=12.35, p=$ .005. Thus, early accelerating throws were judged to be more natural than constant velocity ones for small launch angles and less natural for large launch angles. Early decelerating trials received low ratings for $10^{\circ}$ launch angles and high ratings for the steep angle of $45^{\circ}, F(1,10)=21.24$, $p=.001$. Late acceleration at the end of the trajectory looked much more natural for steep throws than for shallow ones, $F(1,10)=10.50, p=.009$.

Surprisingly, late deceleration was judged to be somewhat more natural than constant velocity trials, $F(1,10)=7.74$, $p=.019$. Perhaps the anticipated stopping of the ball by the catcher made trials that decelerated just before being caught look more natural. Both trajectories with anomalous middle parts were judged less realistic than early accelerating trials: $F(1,10)=30.71, p<.001$, for faster middle parts, and $F(1$, $10)=25.94, p<.001$, for slower midtrajectory speed. The latter slow midtrajectory speeds looked more natural for steep launch angles than for shallow ones, $F(1,10)=16.67$, $p=.002$.

## Discussion

The misconception of postrelease acceleration was mirrored in perceptual judgments. The effect was particularly strong for shallow trajectories. For $10^{\circ}$ launch angles, an early acceleration anomaly was judged to be more natural than the canonical event. Only for steep launch angles of $45^{\circ}$ did observers expect gravity to slow the ball down. This can be taken as evidence for some understanding of the effects of vertical deceleration and acceleration over the course of the trajectory. This effect works against the general conception that the ball should continue to accelerate after it has left the hand and also becomes visible in data showing early deceleration to be tolerated for the steep launch angle but not for the shallow angle.

In the case of large launch angles, observers appeared to mistake horizontal acceleration and deceleration for effects caused by gravity. They attributed horizontal velocity change to causes that could affect only the vertical component: When the magnitude and sign of horizontal velocity changed the way vertical velocity was expected to change caused by gravity, the whole motion was considered to be natural and vice versa. An additional misconception is suggested by the data: Balls that decelerated before they reached the catcher looked comparatively natural. Thus, the anticipation of impending deceleration might also have been reflected in the observers' responses.

## Experiment 5: Animation in Three Dimensions

The frontoparallel plane, on which all simulated balls moved in Experiment 4, is a nongeneric point of view that might produce a particularly strong bias. Moreover, the unicolored stick-figure rendition of the thrower might have been too simplistic to induce a natural scene. Consequently, the graphics were improved and expanded to a 3-D rendition
of a person throwing a ball into a vat. Both were placed on the perspective projection of a square field. Ball trajectories between the frontoparallel and the sagittal plane were introduced, such that the ball would also recede into depth. The number of anomalies that could occur was limited to undue acceleration or deceleration at the beginning or the end of the throw. Again, the vertical motion component remained unchanged in all trials, and the horizontal velocity profiles were changed to produce symmetrical cases, such that anomalous early accelerations would be comparable to late accelerations.

## Method

Participants. Nine observers ( 1 woman and 8 men) participated in the study. All were enrolled as students of various disciplines at the Universitat Bielefeld. They had normal or corrected-to-normal vision and had not participated in any of the previous experiments. They were paid for their participation.
Apparatus and stimuli. The stimuli were created with a Silicon Graphics Personal Indigo2 workstation. The display had a pixel resolution of $1280 \mathrm{H} \times 1024 \mathrm{~V}$ pixels on a $44.8-\mathrm{cm}$ (diagonal) screen with a refresh rate of 72 frames/s. The update rate of the display was 22 Hz . A headrest was used to minimize head motion. The display consisted of a flat checkerboard terrain that was viewed from a position corresponding to a ball-game spectator sitting in one of the cheaper seats fairly high up ( 8.6 m above the ground) and about 30 m away from the playing field. The field receded into depth according to a linear perspective projection. One segmented figure (baseball player) was positioned at the left near corner of the field. This thrower had cone-shaped limbs, such that the recoiling motion necessary for a throw could be simulated using shoulder and elbow joints. A round vat was positioned at the end point of the trajectory such that he would always throw the ball into the vat. The ball was rendered as a polyhedron consisting of 36 vertices and was shaded according to two hidden light sources. Depending on the condition, we placed the vat at the front right comer of the field (frontoparallel throw) or such that the thrower (and the trajectory plane, but not the field) was rotated around his vertical body axis. Thus, in a $45^{\circ}$ rotation case the thrower would face the far right corner of the field. In this case, the vat stood at some distance in front of the corner such that the length of the throw was identical for all rotation conditions.

We were careful to equate the field of view and the player-target distance with the values used in Experiment 4. The thrower subtended 1.8 cm on the projection screen, corresponding to a person who was approximately 1.8 m tall. His simulated distance to the vat was 30.5 m , the distance to the observer was approximately 30 m , and the height of the apex point of the trajectory was 6 m above the ground surface. Viewing distance was 50 cm , such that the display subtended a visual angle of $36^{\circ}$. A throw lasted 2.2 s in the fast condition. Air resistance was assumed to be nonexistent. The following four factors were fully crossed within participants.

1. Average speed. Two horizontal ball velocities were used, $13.7 \mathrm{~m} / \mathrm{s}$ and $9.2 \mathrm{~m} / \mathrm{s}$, that corresponded to durations of the throw between 2.22 and 3.33 s .
2. Type and size of anomaly. The canonical case was supplemented with four anomalous conditions. First, the ball could unduly accelerate, which occurred for the first six frames of the trajectory ( 0.273 ms ) after the ball had left the hand (early acceleration). On each of these frames, the velocity of the ball was multiplied with a factor of 1.1 or 1.3 depending on the size of the anomaly. Second, a deceleration of identical magnitude could occur during the same interval (early deceleration). Third, in a mirror-image fashion, the acceleration anomaly occurred during
the last six frames of the display (late acceleration). Finally, a late deceleration case was created. Gravity was held constant in all trials at $9.8 \mathrm{~m} / \mathrm{s}^{2}$. Thus, only horizontal velocity was manipulated.
3. Direction of throw. All trials started on the left side of the screen and moved toward the right. Rotation values were $0^{\circ}, 22.5^{\circ}$, $45^{\circ}$, and $67.5^{\circ}$; thus, the direction of throw could be either frontoparallel ( $0^{\circ}$ ) or recede into depth. The launch angle of the trajectory with respect to the ground plane (pitch) was always $22.5^{\circ}$.

Procedure. The procedure was similar to that of Experiment 4, but a headrest was used. A short recoiling motion of the thrower's arm preceded the flight phase of the ball. Each throw was shown twice in succession. A naturalness scale, ranging from 0 (unnatural/ strange) to 12 (natural) was then superimposed on the field. Observers had to make a rating by moving the mouse to the appropriate slot. The experimenter mentioned that the thrower was well trained and had no trouble throwing the comparatively large ball (diameter $=0.5-\mathrm{cm}$ screen size when closest to the thrower). It was also emphasized that air resistance was not present. After a practice session of about 20 practice trials representing the complete range of conditions, the experimenter encouraged observers to make use of the whole rating scale and to rely on their intuitive judgment.

## Results

As before, a strong effect of the type of anomaly (acceleration manipulation) was found, $F(4,32)=21.85$, $p<.001$. In contrast to the previous experiment, observers judged the canonical event to be the most natural. Averaged across all other conditions, the anomalous event of the ball accelerating soon after it had left the hand received lower naturalness ratings than the canonical event, $F(1,8)=7.85$, $p=.023$, and higher ratings than all other events ( $8.92<F<20.37, .002<p<.017$ ). Early deceleration was thus judged to be less natural than early acceleration, but more natural than late anomalies, $F(1,8)=13.08, p=$ .007 . No difference was found between late acceleration and late deceleration. As visible in Figure 10 (top panel), the size of the anomaly produced a large main effect, $F(4,32)=$ 43.75, $p<.001$. Larger anomalies looked less natural. Average speed did not produce a significant main effect (see Figure 10, middle panel), but it did interact with the type of anomaly, $F(4,32)=3.40, p=.02$. Fast canonical events were perceived to be the most natural. Direction of throw also did not produce any main effect, but it did interact with type of anomaly; observers differentiated more clearly between canonical and impossible throws when they occurred in the frontoparallel plane, $F(4,32)=21.85, p<$ .001 . This is shown in the bottom panel of Figure 10 for the largest rotation and the frontoparallel case.

An interaction between type and degree of anomaly was also significant, $F(4,32)=2.72, p=.003$. Smaller anomalies tended to look most natural when they consisted in undue acceleration after the ball had left the hand. Early deceleration was detected less reliably but more so than undue acceleration or deceleration toward the end of the descending part of the trajectory. In no case, however, did the canonical event fail to look the most natural. These results need to be qualified by looking at individual performances. Two observers gave the highest naturalness judgments to trials in which the ball accelerated after it left the


Figure 10. Average naturalness ratings by type of anomaly in Experiment 5. The top panel separates the whole data set by the size of the anomaly. Average speed and the two extreme cases for direction of throw are depicted in the middle and bottom panels, respectively. As before, the rating scale ranged from 0 to 12 . Error bars indicate standard errors of the mean.
hand. Two observers correctly identified the canonical events as such and gave all other trials equally poor ratings. The remaining 5 observers judged early accelerating events to be more natural than early decelerating ones. Canonical trials received the highest and late anomalies the lowest naturalness ratings. The latter were never distinguished from one another.

## Discussion

The judgments of throws in directions that did not correspond to the frontoparallel plane differed little from those that did. Thus, the biases could not be attributed to the unnatural viewing situation that was evoked in Experiment 4. Clearly, anomalies early in the trajectory were considered to be more natural than anomalies later in the trajectory, as if the intention to decelerate or accelerate could influence the ball for a while after separation from the hand. A clear anisotropy was found between comparable decelerations and accelerations at the beginning and end of the trajectory.

However, in comparison to Experiment 4, most observers were able to identify the canonical event in the sense that it received the highest naturalness ratings. The size of the anomaly was correlated with the degree of differentiation in the judgments; that is, especially for large anomalies, observers could tell the difference between canonical and anomalous events. Thus, the anomaly size that was chosen might have been too large to go completely unnoticed. The omission of some conditions could also have been responsible for the relatively good resolution compared with Experiment 4. Most likely, however, the launch angle of $22.5^{\circ}$ may have been suggestive of a casual throw. Remember that in Experiment 4 the angle of $10^{\circ}$ produced the strongest bias toward postrelease acceleration. Thus, this bias may reflect the particularly strong force required to make a ball reach its target when thrown at such a shallow angle. This would be consistent with Stimpel's (1933) experiments demonstrating that people have an implicit understanding of the additional force necessary to reach a target once the angle deviates from $45^{\circ}$.

The distance between the position of the thrower and the locus of the anomaly appears to explain the tolerance for unwarranted acceleration or deceleration. Anomalies are tolerated more readily when they happen closer to the thrower. This holds for deceleration as well as for acceleration, with the latter being judged more natural only when it occurs early on. This finding is only partially compatible with impetus theory. The impetus dissipates with time. However, the acquisition of impetus after the ball has left the hand requires a causal nexus that is not severed when the ball leaves the hand. We suggest that it may not only be the intention to accelerate it that is being projected into the ball but also that the actor is perceived to have power over the ball that does not stop at his or her fingertips but rather dissipates gradually over space.

## General Discussion

In five experiments we assessed naive understanding of some neglected but important aspects of projectile motion: trajectory shape and the deceleration and acceleration of vertical and horizontal components of ballistic motion. The pencil-and-paper results obtained in Experiments 1 and 3 reveal surprisingly poor knowledge about object acceleration. About one half of the tested population believed that a thrown ball continued to accelerate after it had left the thrower's hand. The second experiment, in which we presented drawings of a variety of possible and impossible
motion paths, showed that observers were highly tolerant of shape anomalies. Sinusoidal and hyperbolic paths, and to a lesser degree even Puchner's combination of straight ascension followed by a vertical drop, were all judged to be more or less natural paths. Thus, particular preconceptions about projectile paths are not likely to be responsible for the misconception of postrelease acceleration. Although the results are astonishing, Experiment 3 replicated the effect for observers of different experience levels on a different continent.

In Experiment 4 we assessed perceptual knowledge by presenting observers with simulated ballistic projectile motion of natural and anomalous velocity profiles. Observer judgments betrayed similar biases, although they were quantitatively less severe. Among a variety of velocity profiles, shallow trajectories featuring horizontal postrelease acceleration were judged to be more realistic than constant horizontal velocity. This effect was reversed for steep trajectories, indicating that judged naturalness reflects some (implicit) knowledge about the effects of gravity along the vertical dimension. Interestingly, despite this knowledge, throws that decelerated shortly before the ball reached the catcher were judged to be more natural than constant velocity trials. The simulation used for the displays was primitive and limited to a planar two-dimensional view. Thus, in Experiment 5 a subset of conditions was replicated within the context of 3-D perspective rendition of a field. Moreover, the ball could be thrown into the depth of the field. Within the given limited parameter choice, early accelerating trials were no longer the preferred displays. The more realistic 3-D animation of the event put observers in a position to assess the situation more accurately. However, they judged anomalies that occurred close to the observer to be considerably more natural than anomalies occurring at the receiving end. We propose that these results can neither be explained by impetus theory nor by the notion of representational momentum. Instead, observers might misattribute some active force to the ball itself or externalize their body constraints.

## The Limits of Impetus Theory

Medieval impetus theory (McCloskey et al., 1983), Aristotelian physics (Shanon, 1976), and heuristics or patchwork explanations (Kaiser et al., 1992; Krist et al., 1993) have in common that they describe naive reasoning about falling objects as approximations, albeit rough ones, to Newtonian laws of mechanics. They all incorporate the idea of energy conservation, energy consumption, or both. That is, the initial movement continues until it is dissipated by friction (or by exhaustion) or until a new force is applied. Compared with our results, the notion that sometimes the new force erases the old one (impetus theory) is a minor erroneous deviation from Newtonian physics. The finding that objects are thought to accelerate in the absence of an external force is incompatible with all of these models. Our results cannot be explained by the traditional concepts that have been proposed under the umbrella of intuitive physics.

None of these theories has means to allow a force exerted by the arm of the thrower to increase after the object has
spatially separated from the arm. Even an impetus, once transferred to the object, dissipates as it continues to act on the object, such as a curvilinear impetus in the C -shaped tube problem (McCloskey et al., 1980). Moreover, the reversal of effects that seems to occur close to the catcher is altogether incompatible with intuitive physics. A steep ball should just not decelerate before it reaches the catcher. Conceivably, observers grossly overestimated the effects of air resistance and found it plausible to see its cumulated effects toward the end of the trajectory. Such an interpretation would fit with the failure to replicate the late deceleration effect in Experiment 5. In Experiment 5, air resistance was not simulated and all late anomalies were judged to be equally unnatural. Even so, the results do not support notions of impetus theory or any theories in that class, including multidimensional effects as proposed by White (1983).

Our findings are consistent with more basic results that velocity and position are processed separately (Smeets \& Brenner, 1995). Horizontal acceleration and deceleration were sometimes misattributed to gravitational effects acting along the vertical. However, one caveat in interpreting the results as typical or pervasive for projectile perception has to be made. We do not intend to say that performance has reflected the best possible standard. Oudejans, Michaels, Bakker, and Dolne (1996) showed that performance of judging fly balls is best when the observer is in motion trying to catch the ball and is much worse for stationary observers. It likely would be even worse in simulated tasks. Although these and other findings hold for timing and time-to-contact judgments, it is hard to predict how our results would stand up in a different action-oriented context.

## Representational Momentum and Relative Motion

Even simple drawings that depict falling objects or objects without a support do convey dynamic information about the event, and people's immediate memory for position seems to shift over time in the direction of the expected motion (e.g., downward for a falling object). Similarly, an object that halts abruptly is misperceived as being farther ahead in its trajectory. This phenomenon is known in general as "representational momentum" (Bertamini, 1993; Freyd, 1987; Freyd \& Finke, 1984; Freyd \& Pantzer, 1995; Hubbard, 1995). It is possible that in Experiments 1, 2, and 3 the drawings suggested a dynamic interpretation. Maybe observers were prompted to place the point of maximum velocity toward the apex of the trajectory by the fact that they anticipated the position of the ball. Here, as in representational momentum, observers may show a limit in their cognitive ability to deal with changes that happen instantaneously. Rather than an explanation, this is a parallel between two different phenomena, and the differences need to be kept in mind. Although the notion of representational momentum could explain the results of Experiments 1-3, it does not explain the actual misperceptions in Experiments 4 and 5, where representation was not an issue.

When a moving object has a constant velocity, it appears
to accelerate as it moves close to a stationary target. This has been interpreted as a consequence of the fact that relative motion is more salient and its detection threshold is lower than that for absolute motion (Mack, 1986; Smeets \& Brenner, 1994). Such an effect of threshold contrast might explain results with steep launch angles. However, if such an explanation were true, people should always locate the maximum velocity near the thrower because of the stationary reference that the person provides. This explanation cannot account for the perceived deceleration near the catcher. Neither can it explain the clear displacement of maximum perceived velocity away from the thrower in trials with shallow launch angles.

## Externalization of Body Dynamics or Personal Power Gradient

If neither impetus theory nor representational momentum can explain observers' judgments, what guides their understanding of projectile motion? It has been suggested that observers do not have consistent theories at all but that they make up explanations on the fly only when asked for an answer (Cooke \& Breedin, 1994). If this were the case, the question remains, What is it that they make up on the fly? We suggest, that instead of on outward focus on intuitive physics they focus inward on intuitive body dynamics. The powerful notion of an internalization of universal principles (Shepard, 1984, 1994) that govern the physical world can be inverted by proposing that naive participants externalize their body dynamics to make predictions of projectile velocities. Shepard has argued that fundamental and universal structures such as geodesic motion paths have become internalized throughout evolution and now serve to disambiguate stimuli such as apparent motion or imagery of extended body motion. As suggestive as this principle may be, in the particular case of projectile motion, observers might have done the opposite. The action of throwing might have had more evolutionary importance than the prediction of the motion trajectory.

Thus, observers might erroneously have externalized the continuous acceleration of the arm between the recoil movement and the release point of the projectile and projected this continuous acceleration onto the trajectory of the ball after exiting the hand. Likewise, the catching motion might appear as a smooth deceleration of the ball and could have been projected onto the last phase of the ball's motion trajectory in the air. One of the reasons for such an externalization might be that people are often required to anticipate movements of the thrower to detect where he or she is aiming (see Abernethy, 1987). People are also required to anticipate the decelerating catching movements of their hands to avoid injury. Interestingly, we only found high naturalness of late deceleration in Experiment 4, where a receiver was simulated, but not in Experiment 5, where a vat was the target of all throws. Thus, the post hoc notion of externalization of body dynamics can explain the major difference between the two experiments. Externalization might be a natural mechanism that could exert an undesired influence on cognitive as well as on perceptual judgments of projectile velocity.

## Conclusion

If we have reservations about the notion of externalization of body constraints, a somewhat broader explanation of the results could be derived from a general action-oriented perspective. The finding that anomalies occurring close to the actor are judged to be more natural than anomalies farther away could reflect the actor's intention to make corrections toward the end of the accelerating arm movement for better aim. The corrections could, of course, give an extra push or slow the arm down a bit depending on the observer's assessment of the situation. From the perspective of the thrower, who intends to accelerate the ball, it might be a small step to attribute this intention to the ball. And, obviously, if the ball were an animate object, it could easily accelerate in an indefinite number of motion profiles. Such an explanation, however, may conflict with findings that the distinction between animate and inanimate objects is typically made with great accuracy at early ages (Massey \& Gelman, 1988). A more cautious interpretation could dispense with the notion of intentionality or animacy and just consist of the more general assumption of a personal power gradient. Thus, an actor is perceived to exert power over objects, and this power dissipates with distance.

In summary, our findings cannot be explained by the classic contentions of intuitive physics that liken naive understanding of mechanics to Aristotelian or medieval misconceptions. The results are even incompatible with impetus theory. Neither can they be explained by contrast effects that occur in the perception of relative motion. Thus, even if the notion of externalized body dynamics appears to be speculative at this point, the misconceptions reflect, in some sense, intentionality or interference of the related motor action. Our intuitions about ballistic projectile motion seem to be affected by the intended or envisioned motor action. These intuitive biases, within limits, extend to people's perceptual assessment of ballistic motion.

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[^0]:    ${ }^{1}$ There was some uncertainty in the scale of the simulation because gravity rather than familiar size could determine the experienced distances. In this case, one of our independent variables would have the unpleasant side effect of affecting scale. However, observers do not seem to make effective use of gravity to scale their environment (Hecht et al., 1996).

